General covariance of quantum corrections in mini-superspace models

based on the article

Regularization of the cosmological sector of loop quantum gravity with bosonic matter and the related problems with general covariance of quantum corrections
[gr-qc/2001.04491]

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The Quantum Information Structure of Spacetime
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Introduction

Laws of GR should be present in CQGR — the postulated fusion of GR and QFT

Principles of GR concern observers, therefore they should remain classical
(we do not solve the action of diffeomorphism constraint operator on the space of cylindrical functions)

‘Familiar expression’

$$\left\langle \hat{H}^{(\mathrm{gr})} \right\rangle = -\frac{2}{\gamma^2 \kappa} \sum_v N_v \bar{\mathbf{V}}_v \sqrt{\bar{E}^a_1 \delta^1_a \bar{E}^b_2 \delta^2_b \bar{E}^c_3 \delta^3_c}\, \frac{1}{|\bar{E}^d_i \delta^i_d|} \prod_{k \neq i} \frac{\sin\left(\varepsilon_{(k)}\, \bar{A}^k_e \delta^e_k\right)}{\varepsilon_{(k)}} \left[1 + \mathcal{O}\left(\frac{1}{\left(\bar{j}^{(i)}\right)^2} \right) \right]$$

Bosonic matter

Vector field's Hamiltonian

$$H_T^{(\underline{A})} = V^{(\underline{A})} + H^{(\underline{A})} \,,$$

where

$$\begin{aligned} V^{(\underline{A})} &:= \int_{\Sigma_t} d^3x \, N^a \underline{F}_{ab} \underline{E}^b \\ H^{(\underline{A})} &:= \int_{\Sigma_t} d^3x \, N \mathcal{H}^{(\underline{A})} = \frac{\mathrm{g}_{\underline{A}}^2}{2} \int_{\Sigma_t} d^3x \, N \frac{1}{\sqrt{q}} q_{ab} \left(\underline{E}^a \underline{E}^b + \underline{B}^a \underline{B}^b \right) \end{aligned}$$

$$\begin{aligned} \underline{E}^a &:= \frac{\sqrt{q}}{\mathrm{g}_{\underline{A}}^2} e_0^\mu q^{ab} \underline{F}_{\mu b} \\ \underline{B}^a &:= \frac{1}{2\mathrm{g}_{\underline{A}}^2} \epsilon^{abc} \underline{F}_{bc} \end{aligned}$$

Bosonic matter

Scalar field's Hamiltonian

$$H_T^{(\varphi)} = V^{(\varphi)} + H^{(\varphi)} \,,$$

where

$$\begin{aligned} V^{(\varphi)} &:= \int_{\Sigma_t} d^3x \, N^a \partial_a \varphi \, \pi \\ H^{(\varphi)} &:= \frac{1}{2} \int_{\Sigma_t} d^3x \, N \left(\frac{\mathsf{g}_\varphi^2}{\sqrt{q}} \pi^2 + \frac{\sqrt{q}}{\mathsf{g}_\varphi^2} q^{ab} \partial_a \varphi \, \partial_b \varphi \right) \end{aligned}$$

$$\pi := \frac{\sqrt{q}}{\mathsf{g}_\varphi^2} e_0^\mu \partial_\mu \varphi$$

Lattice regularization

Lattice representation of Yang-Mills fields

$$\underline{\mathrm{tr}}\left(\underline{\tau}^I \underline{h}_l\right) = -\frac{\varepsilon}{T}\underline{A}^I_a l^a + \mathcal{O}(\varepsilon^2)$$

$$\underline{\mathrm{tr}}\left(\underline{\tau}^I \underline{h}_{l \circlearrowleft l'}\right) = -\frac{\varepsilon^2}{2T}\underline{F}^I_{ab} l^a l'^b + \mathcal{O}(\varepsilon^4)$$

$$\underline{\mathcal{E}}_I(\mathbf{S}) := \int_{\mathbf{S}} {}^*\underline{E}^I = \int_{\mathbf{S}} n_a \underline{E}^a_I$$

Thiemann's method for regularization of Ashtekar variables

$$\frac{1}{E^a_i}\left(\sqrt{|E|}\right)^n = \frac{2}{n}\frac{\delta \mathbf{V}^n}{\delta E^a_i} = \frac{4}{n\gamma\kappa}\left\{A^i_a, \mathbf{V}^n\right\}$$

$$h^{-1}_{(a)}\left\{h_{(a)}, \mathbf{V}^n\right\} = \varepsilon\left\{A_{(a)}, \mathbf{V}^n\right\} + \mathcal{O}(\varepsilon^2)$$

General covariance of quantum corrections in mini-superspace models

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Lattice-regularized matter contribution to Hamiltonian

Vector field's Hamiltonian

$$H^{(\underline{A})} = \int_{\Sigma_t} d^3x\, N(x) \Big(\mathcal{H}^{(\underline{A})}_{\text{elec}}(x) + \mathcal{H}^{(\underline{A})}_{\text{magn}}(x) \Big)\,,$$

where

$$\begin{aligned}
\mathcal{H}^{(\underline{A})}_{\text{elec}}(x) &= \frac{2^7 \mathbf{g}_{\underline{A}}^2}{(\gamma\kappa)^2} \lim_{\varepsilon\to 0} \underline{E}^a(x)\, \mathrm{tr}\bigg(\tau^i \frac{1}{\varepsilon} h_a^{-1}(x) \Big\{ \mathbf{V}^{\frac{1}{2}}(x), h_a(x) \Big\} \bigg) \\
&\quad \times \int d^3y\, \delta^3(x-y)\, \underline{E}^b(y)\, \mathrm{tr}\bigg(\tau^i \frac{1}{\varepsilon} h_b^{-1}(y) \Big\{ \mathbf{V}^{\frac{1}{2}}(y), h_b(y) \Big\} \bigg) \\
\mathcal{H}^{(\underline{A})}_{\text{magn}}(x) &= \frac{2^7 \mathbf{g}_{\underline{A}}^2}{(\gamma\kappa)^2} \lim_{\varepsilon\to 0} \underline{B}^a(x)\, \mathrm{tr}\bigg(\tau^i \frac{1}{\varepsilon} h_a^{-1}(x) \Big\{ \mathbf{V}^{\frac{1}{2}}(x), h_a(x) \Big\} \bigg) \\
&\quad \times \int d^3y\, \delta^3(x-y)\, \underline{B}^b(y)\, \mathrm{tr}\bigg(\tau^i \frac{1}{\varepsilon} h_b^{-1}(y) \Big\{ \mathbf{V}^{\frac{1}{2}}(y), h_b(y) \Big\} \bigg)
\end{aligned}$$

Lattice-regularized matter contribution to Hamiltonian

Scalar field's Hamiltonian

$$H^{(\varphi)} = \int_{\Sigma_t} d^3x\, N(x) \Big(\mathcal{H}^{(\varphi)}_{\text{mom}}(x) + \mathcal{H}^{(\varphi)}_{\text{der}}(x) + \mathcal{H}^{(\varphi)}_{\text{pot}}(x) \Big) \,,$$

where

$$\begin{aligned}
\mathcal{H}^{(\varphi)}_{\text{mom}}(x) = {} & \frac{2^{21}\,\mathrm{g}_\varphi^2}{3^2(\gamma\kappa)^6} \lim_{\varepsilon\to 0} \pi(x)\, \epsilon_{ijk}\, \epsilon^{abc} \int d^3z\, \delta^3(x-z)\, \mathrm{tr}\bigg(\tau^i \frac{1}{\varepsilon} h_a^{-1}(z) \Big\{ \mathbf{V}^{\frac{1}{2}}(z), h_a(z) \Big\} \bigg) \\
& \times\ \mathrm{tr}\bigg(\tau^j \frac{1}{\varepsilon} h_b^{-1}(z) \Big\{ \mathbf{V}^{\frac{1}{2}}(z), h_b(z) \Big\} \bigg)\, \mathrm{tr}\bigg(\tau^k \frac{1}{\varepsilon} h_c^{-1}(z) \Big\{ \mathbf{V}^{\frac{1}{2}}(z), h_c(z) \Big\} \bigg) \\
& \times \int d^3y\, \delta^3(x-y)\, \pi(y)\, \epsilon_{lmn}\, \epsilon^{def} \int d^3z'\, \delta^3(y-z')\, \mathrm{tr}\bigg(\tau^l \frac{1}{\varepsilon} h_d^{-1}(z') \Big\{ \mathbf{V}^{\frac{1}{2}}(z'), h_d(z') \Big\} \bigg) \\
& \times\ \mathrm{tr}\bigg(\tau^m \frac{1}{\varepsilon} h_e^{-1}(z') \Big\{ \mathbf{V}^{\frac{1}{2}}(z'), h_e(z') \Big\} \bigg)\, \mathrm{tr}\bigg(\tau^n \frac{1}{\varepsilon} h_f^{-1}(z') \Big\{ \mathbf{V}^{\frac{1}{2}}(z'), h_f(z') \Big\} \bigg) \\
\mathcal{H}^{(\varphi)}_{\text{der}}(x) = {} & \frac{2^{17}}{3^4(\gamma\kappa)^4\,\mathrm{g}_\varphi^2} \lim_{\varepsilon\to 0} \partial_a\varphi(x)\, \epsilon_{ijk}\, \epsilon^{abc}\, \mathrm{tr}\bigg(\tau^j \frac{1}{\varepsilon} h_b^{-1}(x) \Big\{ \mathbf{V}^{\frac{3}{4}}(x), h_b(x) \Big\} \bigg)\, \mathrm{tr}\bigg(\tau^k \frac{1}{\varepsilon} h_c^{-1}(x) \Big\{ \mathbf{V}^{\frac{3}{4}}(x), h_c(x) \Big\} \bigg) \\
& \times \int d^3y\, \delta^3(x-y)\, \partial_d\varphi(y)\, \epsilon_{ilm}\, \epsilon^{def}\, \mathrm{tr}\bigg(\tau^l \frac{1}{\varepsilon} h_e^{-1}(y) \Big\{ \mathbf{V}^{\frac{3}{4}}(y), h_e(y) \Big\} \bigg)\, \mathrm{tr}\bigg(\tau^m \frac{1}{\varepsilon} h_f^{-1}(y) \Big\{ \mathbf{V}^{\frac{3}{4}}(y), h_f(y) \Big\} \bigg) \\
\mathcal{H}^{(\varphi)}_{\text{pot}}(x) = {} & \frac{1}{2\,\mathrm{g}_\varphi^2} \sqrt{q(x)}\, \mathcal{V}[\varphi(x)] \approx \frac{1}{2\,\mathrm{g}_\varphi^2} \lim_{\varepsilon\to 0} \mathcal{V}[\varphi(x)]\, \frac{1}{\varepsilon^3} \mathbf{V}(x,\varepsilon)
\end{aligned}$$

Kinematics

States space

$$\mathscr{H}_{kin} \ni |\Gamma; m_l, \underline{m}_l, U_\pi\rangle := |\Gamma; m_l\rangle \otimes |\Gamma; \underline{m}_l\rangle \otimes |\Gamma; U_\pi\rangle$$

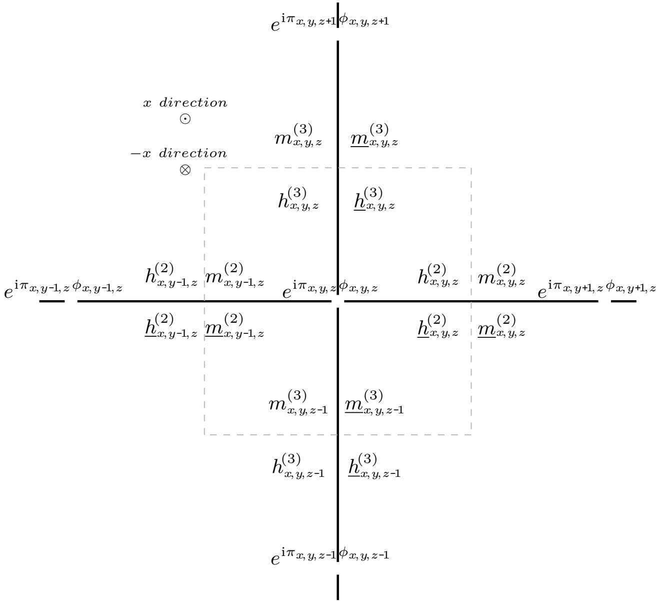
Cosmological, node-centered coherent states

$$|\ \rangle = |\bar{\ }\rangle^{(\mathrm{gr})} \otimes |\ \rangle^{(\mathrm{matt})} = |\bar{\ }\rangle^{(\mathrm{gr})} \otimes \bigotimes_{\boldsymbol{\phi}} |\ \rangle^{(\boldsymbol{\phi})}$$

Normalized basis states

$$\left|\bar{m}_v^{(i)}\right\rangle := \left|\frac{1}{2} m^{(i)}_{v-\varepsilon^{(i)}}, \frac{1}{2} m_v^{(i)}\right\rangle = \exp\left[\frac{\mathrm{i}}{2}\left(m^{(i)}_{v-\varepsilon^{(i)}} + m_v^{(i)}\right) c^{(i)}\right] = \exp\left[\mathrm{i}\bar{m}_v^{(i)} c^{(i)}\right]$$

Kinematics


$e^{\mathrm{i}\pi_{x,y,z+1}}\phi_{x,y,z+1}$
x direction ⊙
$-x$ direction ⊗
$m^{(3)}_{x,y,z}$
$\underline{m}^{(3)}_{x,y,z}$
$h^{(3)}_{x,y,z}$
$\underline{h}^{(3)}_{x,y,z}$
$h^{(2)}_{x,y-1,z}$
$m^{(2)}_{x,y-1,z}$
$h^{(2)}_{x,y,z}$
$m^{(2)}_{x,y,z}$
$e^{\mathrm{i}\pi_{x,y-1,z}}\phi_{x,y-1,z}$
$e^{\mathrm{i}\pi_{x,y,z}}\phi_{x,y,z}$
$e^{\mathrm{i}\pi_{x,y+1,z}}\phi_{x,y+1,z}$
$\underline{h}^{(2)}_{x,y-1,z}$
$\underline{m}^{(2)}_{x,y-1,z}$
$\underline{h}^{(2)}_{x,y,z}$
$\underline{m}^{(2)}_{x,y,z}$
$m^{(3)}_{x,y,z-1}$
$\underline{m}^{(3)}_{x,y,z-1}$
$h^{(3)}_{x,y,z-1}$
$\underline{h}^{(3)}_{x,y,z-1}$
$e^{\mathrm{i}\pi_{x,y,z-1}}\phi_{x,y,z-1}$

Kinematics

Reduced Ashtekar variables

$$\bar{A}^i_a(t) := A^{(i)}_{(a)}(t)\,{}^0e^i_a = \frac{1}{\varepsilon}c^{(i)}(t)$$
$$\bar{E}^a_i(t) := E^{(a)}_{(i)}(t)\,{}^0e^a_i = \frac{1}{\varepsilon^2}p^{(i)}(t)$$

Reduced holonomy

$$h^{(i)}(v) = \exp\left(\int_0^{\varepsilon} ds\,\bar{A}^i_a \tau^i l^a_\nu(s)\right) = e^{c^i \tau^i}$$

Actions of lattice-regularized canonical variables on the normalized states

$$\hat{c}^{(i)}\left|\bar{m}^{(i)}_v\right\rangle = \frac{\mathrm{i}}{\varepsilon}\left(\left|\bar{m}^{(i)}_{v-\varepsilon^{(i)}} - \frac{\varepsilon}{2}\right\rangle - \left|\bar{m}^{(i)}_v + \frac{\varepsilon}{2}\right\rangle\right)$$
$$\hat{p}^{(i)}\left|\bar{m}^{(i)}_v\right\rangle = \bar{m}^{(i)}_v \bar{k}\left|\bar{m}^{(i)}_v\right\rangle, \qquad \bar{k} := \frac{1}{2}\gamma\hbar\kappa = 8\pi\gamma l_P^2$$

Coherent states

Normalized, node-centered coherent states

$$\Big|\,\bar{\ }\,\Big\rangle^{(\mathrm{gr})} := \sum_{v} \bigotimes_{i}^{3} \left(\left\langle \bar{\mathfrak{c}}_v^{(i)}(A) \middle| \bar{\mathfrak{c}}_v^{(i)}(A) \right\rangle \right)^{-\frac{1}{2}} \left| \bar{\mathfrak{c}}_v^{(i)}(A) \right\rangle$$

The link-oriented, node-centered shadow state

$$\left| \bar{\mathfrak{c}}_v^{(i)}(A) \right\rangle := \sum_{\bar{m}_v^{(i)}} \exp\left[-\frac{1}{2d^2} \left(\bar{m}_v^{(i)} - \frac{p^{(i)}}{\hbar} \right)^2 \right] \exp\left[-\mathrm{i} \left(\bar{m}_v^{(i)} - \frac{p^{(i)}}{\hbar} \right) c^{(i)} \right] \left| \bar{m}_v^{(i)} \right\rangle$$

Relation with standard LQC

$$\bar{m}_v^{(i)} \to \frac{1}{2} \mu_v^{(i)}$$

Coherent states

Correspondence principle

$$\bar{m}_v^{(i)} \to \frac{p^{(i)}}{\not{k}}$$

Expectation value of the reduced connection operator

$$\left\langle \hat{c}^{(i)} \right\rangle = c^{(i)} \Big(1 + \mathcal{O}\big(\varepsilon^2\big) \Big)$$

Expectation value of the reduced flux operator

$$\left\langle \hat{p}^{(i)} \right\rangle = p^{(i)}$$

Ehrenfest theorem

Semiclassical dynamics of the whole system

$$\frac{d\left\langle \hat{c}\right\rangle^{(\mathrm{gr})}}{dt} = \frac{1}{\mathrm{i}\hbar}\left(\left\langle\left[\hat{c},\hat{H}^{(\mathrm{gr})}\right]\right\rangle^{(\mathrm{gr})} + \left\langle\left[\hat{c},\hat{H}^{(\mathrm{matt})}\right]\right\rangle^{(\mathrm{gr})}\right) = \frac{1}{\mathrm{i}\hbar}\left\langle\left[\hat{c},\hat{H}^{(\mathrm{gr})}\right]\right\rangle^{(\mathrm{gr})} + \Delta_c^{H^{(\mathrm{matt})}}$$

$$\frac{dp}{dt} = \frac{1}{\mathrm{i}\hbar}\left\langle\left[\hat{p},\hat{H}^{(\mathrm{gr})}\right]\right\rangle^{(\mathrm{gr})}$$

$$\frac{d\left\langle \hat{\boldsymbol{\phi}}\right\rangle^{(\mathrm{matt})}}{dt} = \frac{1}{\mathrm{i}\hbar}\left\langle\left\langle\left[\hat{\boldsymbol{\phi}},\hat{H}^{(\mathrm{matt})}\right]\right\rangle^{(\mathrm{matt})}\right\rangle^{(\mathrm{gr})} = \frac{1}{\mathrm{i}\hbar}\left\langle\left[\hat{\boldsymbol{\phi}},\hat{H}^{(\mathrm{matt})}\right]\right\rangle^{(\mathrm{matt})} + \Delta_{\boldsymbol{\phi}}^{|\rangle^{(\mathrm{gr})}}$$

Quantum-gravitational corrections, $\delta_{\dot{c}} \propto \delta_c \propto \delta_p \propto \varepsilon^2$

$$\frac{d\left\langle \hat{c}\right\rangle^{(\mathrm{gr})}}{dt} = \frac{dc}{dt}\left(1+\delta_{\dot{c}}\right), \qquad \left\langle\left[\hat{c},\hat{H}^{(\mathrm{gr})}\right]\right\rangle^{(\mathrm{gr})} = \mathrm{i}\hbar\frac{\delta H^{(\mathrm{gr})}}{\delta p}\left(1+\delta_c\right)$$

$$\left\langle\left[\hat{p},\hat{H}^{(\mathrm{gr})}\right]\right\rangle^{(\mathrm{gr})} = -\,\mathrm{i}\hbar\frac{\delta H^{(\mathrm{gr})}}{\delta c}\left(1+\delta_p\right)$$

Ehrenfest theorem

Source of quantum generally-relativistic corrections, $\Delta_c^{H^{(\mathrm{matt})}} \propto \Delta_{\boldsymbol{\phi}}^{|\,\rangle^{(\mathrm{gr})}} \propto 1/\bar{m}^2$

$$\left\langle \left\langle \left[\hat{\boldsymbol{\phi}}, \hat{H}^{(\mathrm{matt})} \right] \right\rangle^{(\mathrm{matt})} \right\rangle^{(\mathrm{gr})} = \left\langle \left[\hat{\boldsymbol{\phi}}, \left\langle \hat{H}^{(\mathrm{matt})} \right\rangle^{(\mathrm{gr})} \right] \right\rangle^{(\mathrm{matt})}$$

Corrections to dynamics of the vector field

$$\left\langle \left\langle \left[\hat{\boldsymbol{\phi}}, \hat{H}^{(\underline{A})} \right] \right\rangle^{(\underline{A})} \right\rangle^{(\mathrm{gr})} = \sum_a \left\langle \left\langle \left[\hat{\boldsymbol{\phi}}, \hat{H}^{(\underline{A})}_{(a)} \right] \right\rangle^{(\underline{A})} \right\rangle^{(\mathrm{gr})} = \sum_a \left\langle \left[\hat{\boldsymbol{\phi}}, \hat{H}^{(\underline{A})}_{(a)} \right] \right\rangle^{(\underline{A})} \left(1 + \delta^{(\underline{A})}_{(a)} \right)$$

Corrections to dynamics of the scalar field

$$\left\langle \left\langle \left[\hat{\boldsymbol{\phi}}, \hat{H}^{(\varphi)} \right] \right\rangle^{(\varphi)} \right\rangle^{(\mathrm{gr})} = \left\langle \left[\hat{\boldsymbol{\phi}}, \hat{H}^{(\varphi)}_{\mathrm{mom}} \right] \right\rangle^{(\varphi)} \left(1 + \delta^{(\varphi)}_{\mathrm{mom}} \right) + \sum_a \left\langle \left[\hat{\boldsymbol{\phi}}, \hat{H}^{(\varphi)}_{(a)_{\mathrm{der}}} \right] \right\rangle^{(\varphi)} \left(1 + \delta^{(\varphi)}_{(a)_{\mathrm{der}}} \right) + \left\langle \left[\hat{\boldsymbol{\phi}}, \hat{H}^{(\varphi)}_{\mathrm{pot}} \right] \right\rangle^{(\varphi)}$$

Ehrenfest theorem

Action of the quantum generally-relativistic corrections-generating operator

$$\mathrm{tr}\left(\tau^{(i)}\hat{h}_{(j)}^{-1}\left[\hat{\mathbf{V}}^{n},\hat{h}_{(j)}\right]\right)\bigotimes_{k}^{3}\left|\bar{m}_{v}^{(k)}\right\rangle=\frac{\mathrm{i}\,n}{4\bar{m}_{v}^{(i)}}\left(\bar{k}^{3}\,\bar{m}_{v}^{1}\,\bar{m}_{v}^{2}\,\bar{m}_{v}^{3}\right)^{\frac{n}{2}}\delta_{(j)}^{(i)}$$

$$\times\left[1+\frac{n^{2}-6n+8}{2^{5}\cdot 3}\frac{1}{\left(\bar{m}_{v}^{(i)}\right)^{2}}+\mathcal{O}\left(\frac{1}{\left(\bar{m}_{v}^{(i)}\right)^{4}}\right)\right]\bigotimes_{k}^{3}\left|\bar{m}_{v}^{(k)}\right\rangle,$$

where $\hat{\hat{\mathbf{V}}}$ was expanded around the coherent state

$$\hat{\hat{\mathbf{V}}}^{n}=\varepsilon^{3n}\left(\langle\hat{\hat{q}}\rangle\right)^{\frac{n}{2}}\sum_{k=0}^{\infty}\binom{n/2}{k}\left(\frac{\hat{\hat{q}}-\langle\hat{\hat{q}}\rangle}{\langle\hat{\hat{q}}\rangle}\right)^{k},$$

$$\langle\hat{\hat{q}}\rangle=\bar{k}^{3}\,\bar{m}^{1}\bar{m}^{2}\bar{m}^{3}$$

Ehrenfest theorem

Explicit value of the dimensionless corrections

$$\delta_{(i)}^{(\underline{A})} = \frac{7}{2^6} \frac{1}{\left(\bar{m}_v^{(i)}\right)^2} \to 7\pi^2\gamma^2 \frac{l_P^4}{\varepsilon^4} \frac{q_{(i)(i)}}{|\bar{q}|}$$

$$\delta_{\mathrm{mom}}^{(\varphi)} = \frac{7}{2^6} \sum_i^3 \frac{1}{\left(\bar{m}_v^{(i)}\right)^2} = \sum_i^3 \delta_{(i)}^{(\underline{A})}$$

$$\delta_{(i)\,\mathrm{der}}^{(\varphi)} = \frac{65}{2^8 \cdot 3} \sum_{j \neq i} \frac{1}{\left(\bar{m}_v^{(j)}\right)^2} = \frac{65}{84} \sum_{j \neq i} \delta_{(j)}^{(\underline{A})}$$

QFT and GR perspective

QFT — ontological perspective

States and operators are beyond the measurable and observable world

GR — epistemological perspective

From the perspective of the astronomy and experimental physics, we can only 'sense' observables

[Strong] formulation of equivalence principle

The **outcome of any local, nongravitational** *[or gravitational]* **test experiment** *is independent of the experimental apparatus' velocity relative to the gravitational field and is independent of where and when in the gravitational field the experiment is performed.*

Cited after M. P. Haugan and C. Lämmerzahl, Lect. Notes Phys. **562**, 195 (2001) [gr-qc/0103067]

QFT and GR perspective

Restrictions on the formulation of canonical quantum general relativity (CQGR):

a) [QFT] Quantization is performed in a canonical procedure

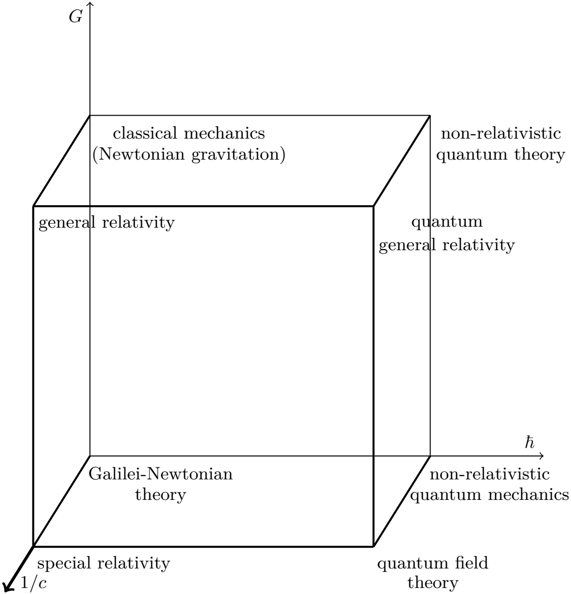
b) [QFT] Semiclassical limit reproduces measurable observables

c) [QFT/GR] Gauge symmetries are preserved

d) [GR] Equivalence principle is satisfied

e) [GR] Theory is background independent

QFT and GR perspective



QFT and GR perspective

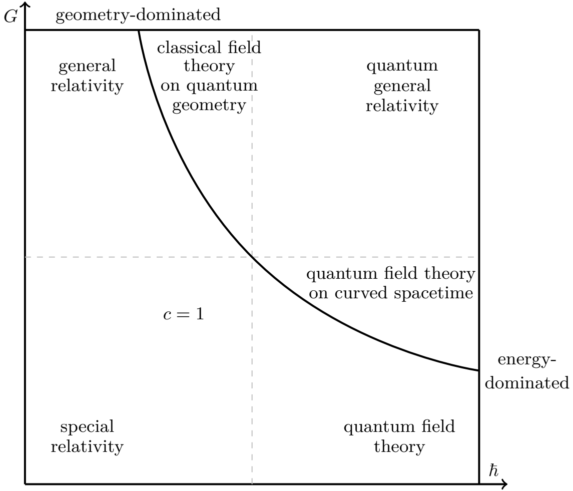

G
geometry-dominated
general relativity
classical field theory on quantum geometry
quantum general relativity
quantum field theory on curved spacetime
$c = 1$
energy-dominated
special relativity
quantum field theory
$\hbar$

Figure: Planck hyperbola, $l_P =$ constant

QFT and GR perspective

$$\begin{tikzcd}
 & \text{quantum field theory on curved spacetime} \arrow[dr, "\hbar\to 0"] & \\
\text{quantum general relativity} \arrow[ur, "G\gtrsim 0"] \arrow[dr, "\hbar\gtrsim 0"'] & \xrightarrow{l_P\to 0} & \text{special relativity} \\
 & \text{classical field theory on quantum geometry} \arrow[ur, "G\to 0"'] &
\end{tikzcd}$$

Quantum generally-relativistic corrections

Respecting general covariance in the vector field's sector

$$\left\langle \hat{H}^{(\underline{A})} \right\rangle = \sum_{i}^{3} \left\langle \hat{H}_{(i)}^{(\underline{A})} \right\rangle = \sum_{i}^{3} \left\langle \hat{H}_{(i)}^{(\underline{A})} \right\rangle^{(\underline{A})} \left(1 + \delta_{(i)}^{(\underline{A})} \right) .$$

Electric vector and magnetic pseudovector **densities** — the physical modes of the electromagnetic wave

Quantum generally-relativistic corrections

Violation of general covariance in the scalar field's sector

$$\begin{aligned}\left\langle\hat{H}^{(\varphi)}\right\rangle &= \sum_{i}^{3}\left[\left\langle\hat{H}^{(\varphi)}_{(i)\,\mathrm{mom}}\right\rangle + \left\langle\hat{H}^{(\varphi)}_{(i)\,\mathrm{der}}\right\rangle + \left\langle\hat{H}^{(\varphi)}_{(i)\,\mathrm{pot}}\right\rangle\right]\\ &= \sum_{i}^{3}\left[\left\langle\hat{H}^{(\varphi)}_{(i)\,\mathrm{mom}}\right\rangle^{(\varphi)}\left(1+2\delta^{(\underline{A})}_{(i)}\right) + \left\langle\hat{H}^{(\varphi)}_{(i)\,\mathrm{der}}\right\rangle^{(\varphi)}\left(1-\delta^{(\underline{A})}_{(i)}+\frac{65}{84}\sum_{j\neq i}\delta^{(\underline{A})}_{(j)}\right)\right.\\ &\quad\left. + \left\langle\hat{H}^{(\varphi)}_{(i)\,\mathrm{pot}}\right\rangle^{(\varphi)}\left(1-\delta^{(\underline{A})}_{(i)}\right)\right]\left(1+\delta^{(\underline{A})}_{(i)}\right) + \mathcal{O}\left(\left(\delta^{(\underline{A})}_{(i)}\right)^{2}\right),\end{aligned}$$

where

$$\left\langle\hat{H}^{(\varphi)}_{1\,\mathrm{mom}}\right\rangle = \left\langle\hat{H}^{(\varphi)}_{2\,\mathrm{mom}}\right\rangle = \left\langle\hat{H}^{(\varphi)}_{3\,\mathrm{mom}}\right\rangle = \frac{1}{3}\left\langle\hat{H}^{(\varphi)}_{\mathrm{mom}}\right\rangle$$

and

$$\left\langle\hat{H}^{(\varphi)}_{1\,\mathrm{pot}}\right\rangle = \left\langle\hat{H}^{(\varphi)}_{2\,\mathrm{pot}}\right\rangle = \left\langle\hat{H}^{(\varphi)}_{3\,\mathrm{pot}}\right\rangle = \frac{1}{3}\left\langle\hat{H}^{(\varphi)}_{\mathrm{pot}}\right\rangle$$

What are the scalar field's observables?

Covarianization

Fixing the total energy of a considered system

$$H = 0$$

Hamiltonian in minisuperspace models

$$\bar{H} = \sum_{i}^{3} \left(\bar{H}^{(\mathrm{gr})}_{(i)} + \bar{H}^{(\underline{A})}_{(i)} + \bar{H}^{(\varphi)}_{(i)} \right) =: \sum_{i}^{3} \bar{H}_{(i)}$$

Scalar field represented by the isotropic Proca Hamiltonian
(associated with Lorenz gauge condition — a Lorentz invariant condition)

Procedure of the covarianization

$$\bar{H}_{(i)} \xrightarrow{\mathrm{covar.}} \bar{H}_{(i)} \left(1 + \delta^{(\underline{A})}_{(i)} \right)^{-1} .$$

Mass term, or any other additional potential is **shifted down** by $\sum_{i}^{3} \left(1 - \delta^{(\underline{A})}_{(i)} \right)$

Quantum generally-relativistic corrections

Remarkable result in the fermionic sector

$$
\begin{aligned}
H^{(\psi)} = \int_{\Sigma_t} d^3x \, \frac{N}{\sqrt{q}} \Bigg[& \epsilon_{ijk} \, \epsilon^{abe} \, \mathrm{tr}\left(\tau^i \frac{1}{\varepsilon} h_c^{-1}(x) \left\{ \mathbf{V}^{\frac{1}{2}}(x), h_c(x) \right\} \right) \\
& \times \, \mathrm{tr}\left(\tau^j \frac{1}{\varepsilon} h_d^{-1}(x) \left\{ \mathbf{V}^{\frac{1}{2}}(x), h_d(x) \right\} \right) (\mathrm{fermionic})_e^k \Bigg] \delta_a^c \delta_b^d \, ,
\end{aligned}
$$

where

$$
(\mathrm{fermionic})_e^k \propto \mathcal{D}_e \mathcal{J}^k(\mathbf{x}) - \big(\pi_\xi(\mathbf{x}) \sigma^k \mathcal{D}_e \xi(\mathbf{x}) - \pi_\chi(\mathbf{x}) \sigma^k \mathcal{D}_e \chi(\mathbf{x}) + \mathrm{c.c.} \big) - \frac{1}{\gamma} A_e^k(\mathbf{x}) \mathcal{J}^0(\mathbf{x})
$$

Antisymmetry in the spatial indices

Covarianization

$$
\begin{aligned}
H = \int_{\Sigma_t} d^3x\, N \Bigg[& \frac{1}{\kappa\sqrt{q}} \Big(\big(F^i_{cd} - (\gamma^2+1)\epsilon_{ilm} K^l_c K^m_d \big) \epsilon^{ijk} E^a_j E^b_k \Big) + \frac{\mathsf{g}_{\underline{A}}^2}{2\sqrt{q}}\, q_{cd} \big(\underline{E}^a_I \underline{E}^b_I + \underline{B}^a_I \underline{B}^b_I \big) \\
& + \frac{2\sqrt{q}}{3\kappa} \Lambda\, q^{ab} q_{cd} + \mathcal{H}^{(\varphi)\,ab}\, q_{cd} + \mathcal{H}^{(\psi)\,ab}_{cd} \Bigg] \delta^c_a \delta^d_b =: H^{ab}_{cd}\, \delta^c_a \delta^d_b
\end{aligned}
$$

Covarianization (when restricted to the leading order corrections)

$$
H^{ab}_{cd}(v) \overset{\text{covar.}}{\longrightarrow} H^{ab}_{cd}(v) \left(1 + \frac{1}{2} \delta^{(\underline{A})}_{(a)}(v) \right)^{-1} \left(1 + \frac{1}{2} \delta^{(\underline{A})}_{(b)}(v) \right)^{-1}
$$

(+) Aftermaths occur accordingly in the gravitational and cosmological constant sector, as well as in all the potential terms
(−) **Local** deformations to the already deformed (from an observer's perspective) diffeomorphism symmetry

Covarianization

Added to the Thiemann's regularization method when defining the regularization of the matter degrees of freedom

Isotropic limit of the covarianization

$$H \xrightarrow{\text{covar.}} H\left(1+\bar{\delta}^{(\underline{A})}\right)^{-1}, \qquad \bar{\delta}^{(\underline{A})} := \frac{1}{3}\sum_{a}^{3} \delta^{(\underline{A})}_{(a)}$$

In homogeneous and isotropic LQC is possible to formulate QFT on the effective 'covarianized background'

Conclusions

- Quantum generally-relativistic corrections reflect ‘semi-flat’ approximation of the classical field theory on quantum geometry (bounce-resolving formalism)
- Results of our analysis are valid for any ADM spacetime symmetry and easily generalizable to any internal symmetries
- Densitized, link-oriented variables lead to ‘well-behaving’ corrections
- Point holonomy regularization is disfavored
- Covarianization restores general covariance (the Einstein equivalence principle), deforming diffeomorphism invariance back to the observer’s perspective
- Covarianization modifies the inverse volume corrections in the gravitational sector and shifts all potential terms in the matter sector
- Discussion concerning the locality issue related to the covarianization is welcome
- All the details are presented in [gr-qc/2001.04491]